EXERCISE 5
MODULAR FORMS 2019
DUE DATE: MAY 19, 2019

Exercise 1. Let $L \subset \mathbb{R}^{24}$ be an even, self-dual lattice in dimension 24 that has no vectors of norm $|\ell|^2 = 2$ (the Leech lattice is the unique example). Find the number of vectors of norm $|\ell|^2 = 4$.

Exercise 2. We defined the even self-dual (unimodular) lattices $E(8m) \subset \mathbb{R}^{8m}$ as the lattice generated by the vector $\delta = (\frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{2})$ and the lattice $F = \{x \in \mathbb{Z}^{8m} : \sum_{j=1}^{8m} x_j = 0 \bmod 2\}$.

Show that for any $m \geq 2$, the vectors of norm $|\ell|^2 = 2$ in $E(8m)$ are precisely the vectors $\pm e_i \pm e_k$, $1 \leq i < k \leq 8m$ where e_i are the standard basis vectors: $e_1 = (1, 0, \ldots, 0)$ etc.